



Oxford Cambridge and RSA

**Monday 16 May 2022 – Afternoon**

**AS Level Further Mathematics A**

**Y531/01 Pure Core**

**Time allowed: 1 hour 15 minutes**



**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- This document has **4** pages.

**ADVICE**

- Read each question carefully before you start your answer.

Answer **all** the questions.

- 1 (a) Determine whether the point  $(19, -12, 17)$  lies on the line  $\mathbf{r} = \begin{pmatrix} 4 \\ -2 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$ . [3]

Vectors  $\mathbf{a}$  and  $\mathbf{b}$  are given by  $\mathbf{a} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} -3 \\ 6 \\ 2 \end{pmatrix}$ .

- (b) (i) Find, in degrees, the angle between  $\mathbf{a}$  and  $\mathbf{b}$ . [3]

- (ii) Find a vector which is perpendicular to both  $\mathbf{a}$  and  $\mathbf{b}$ . [2]

- 2 Matrices  $\mathbf{A}$  and  $\mathbf{B}$  are given by  $\mathbf{A} = \begin{pmatrix} a & 1 \\ -1 & 3 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} -2 & 5 \\ -1 & 0 \end{pmatrix}$  where  $a$  is a constant.

- (a) Find the following matrices.

- $\mathbf{A} + \mathbf{B}$
- $\mathbf{AB}$
- $\mathbf{A}^2$  [3]

- (b) (i) Given that the determinant of  $\mathbf{A}$  is 25 find the value of  $a$ . [2]

- (ii) You are given instead that the following system of equations does **not** have a unique solution.

$$ax + y = -2$$

$$-x + 3y = -6$$

- Determine the value of  $a$ . [2]

- 3 **In this question you must show detailed reasoning.**

The roots of the equation  $5x^3 - 3x^2 - 2x + 9 = 0$  are  $\alpha$ ,  $\beta$  and  $\gamma$ .

Find a cubic equation with integer coefficients whose roots are  $\alpha\beta$ ,  $\beta\gamma$  and  $\gamma\alpha$ . [6]

- 4 Prove that  $3^n > 10n$  for all integers  $n \geq 4$ . [5]

**5 In this question you must show detailed reasoning.**

(a) Use an algebraic method to find the square roots of  $-16 + 30i$ . [5]

(b) By finding the cube of one of your answers to part (a) determine a cube root of  $\frac{-99 + 5i}{4}$ .

Give your answer in the form  $a + bi$ . [2]

**6** The matrix **A** is given by  $\mathbf{A} = \frac{1}{13} \begin{pmatrix} 5 & 12 \\ 12 & -5 \end{pmatrix}$ .

You are given that **A** represents the transformation **T** which is a reflection in a certain straight line. You are also given that this straight line, the mirror line, passes through the origin,  $O$ .

(a) Explain why there must be a line of invariant points for **T**. State the geometric significance of this line. [2]

(b) By considering the line of invariant points for **T**, determine the equation of the mirror line. Give your answer in the form  $y = mx + c$ . [4]

The coordinates of the point  $P$  are  $(1, 5)$ .

(c) By considering the image of  $P$  under the transformation **T**, or otherwise, determine the coordinates of the point on the mirror line which is closest to  $P$ . [3]

(d) The line with equation  $y = ax + 2$  is an invariant line for **T**.

Determine the value of  $a$ . [2]

**7 In this question you must show detailed reasoning.**

Two loci,  $C_1$  and  $C_2$ , are defined as follows.

$$C_1 = \left\{ z : \arg(z + 2 - i) = \frac{1}{4}\pi \right\} \quad \text{and} \quad C_2 = \left\{ z : \arg(z - 2 - \sqrt{3} - 2i) = \frac{2}{3}\pi \right\}$$

By considering the representations of  $C_1$  and  $C_2$  on an Argand diagram, determine the locus  $C_1 \cap C_2$ . [7]

**Turn over for question 8**

8 The line segment  $AB$  is a diameter of a sphere,  $S$ . The point  $C$  is **any** point on the surface of  $S$ .

(a) Explain why  $\vec{AC} \cdot \vec{BC} = 0$  for **all** possible positions of  $C$ . [3]

You are now given that  $A$  is the point  $(11, 12, -14)$  and  $B$  is the point  $(9, 13, 6)$ .

(b) Given that the coordinates of  $C$  have the form  $(2p, p, 1)$ , where  $p$  is a constant, determine the coordinates of the possible positions of  $C$ . [6]

**END OF QUESTION PAPER**

---

**OCR**

Oxford Cambridge and RSA

**Copyright Information**

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website ([www.ocr.org.uk](http://www.ocr.org.uk)) after the live examination series. If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.