Oxford Cambridge and RSA

Monday 16 May 2022 – Afternoon

AS Level Further Mathematics A

Y531/01 Pure Core

Time allowed: 1 hour 15 minutes

9 4 5 4 6 6 4 9 4 3

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **4** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 **(a)** Determine whether the point (19, −12, 17) lies on the line $\mathbf{r} = \begin{pmatrix} 4 \\ -2 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$. **[3]**

Vectors **a** and **b** are given by $\mathbf{a} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -3 \\ 6 \\ 2 \end{pmatrix}$.

(b) **(i)** Find, in degrees, the angle between **a** and **b**. **[3]**

(ii) Find a vector which is perpendicular to both **a** and **b**. **[2]**

2 Matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} a & 1 \\ -1 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -2 & 5 \\ -1 & 0 \end{pmatrix}$ where a is a constant.

(a) Find the following matrices.

- $\mathbf{A} + \mathbf{B}$
- $\mathbf{AB}$
- $\mathbf{A}^2$

[3]

(b) **(i)** Given that the determinant of **A** is 25 find the value of a. **[2]**

(ii) You are given instead that the following system of equations does **not** have a unique solution.

$$ax + y = -2$$
$$-x + 3y = -6$$

Determine the value of a. **[2]**

3 **In this question you must show detailed reasoning.**

The roots of the equation $5x^3 - 3x^2 - 2x + 9 = 0$ are α, β and γ.

Find a cubic equation with integer coefficients whose roots are $\alpha\beta$, $\beta\gamma$ and $\gamma\alpha$. **[6]**

4 Prove that $3^n > 10n$ for all integers $n \geqslant 4$. **[5]**

5 **In this question you must show detailed reasoning.**

(a) Use an algebraic method to find the square roots of $-16 + 30\text{i}$. **[5]**

(b) By finding the cube of one of your answers to part **(a)** determine a cube root of $\frac{-99+5\text{i}}{4}$.

Give your answer in the form $a + b\text{i}$. **[2]**

6 The matrix **A** is given by $\mathbf{A} = \frac{1}{13}\begin{pmatrix} 5 & 12 \\ 12 & -5 \end{pmatrix}$.

You are given that **A** represents the transformation T which is a reflection in a certain straight line. You are also given that this straight line, the mirror line, passes through the origin, O.

(a) Explain why there must be a line of invariant points for T. State the geometric significance of this line. **[2]**

(b) By considering the line of invariant points for T, determine the equation of the mirror line. Give your answer in the form $y = mx + c$. **[4]**

The coordinates of the point P are (1, 5).

(c) By considering the image of P under the transformation T, or otherwise, determine the coordinates of the point on the mirror line which is closest to P. **[3]**

(d) The line with equation $y = ax + 2$ is an invariant line for T.

Determine the value of a. **[2]**

7 **In this question you must show detailed reasoning.**

Two loci, C_1 and C_2, are defined as follows.

$C_1 = \left\{z : \arg(z+2-\text{i}) = \frac{1}{4}\pi\right\}$ and $C_2 = \left\{z : \arg(z-2-\sqrt{3}-2\text{i}) = \frac{2}{3}\pi\right\}$

By considering the representations of C_1 and C_2 on an Argand diagram, determine the locus $C_1 \cap C_2$. **[7]**

Turn over for question 8

8 The line segment AB is a diameter of a sphere, S. The point C is **any** point on the surface of S.

(a) Explain why $\overrightarrow{AC} \cdot \overrightarrow{BC} = 0$ for **all** possible positions of C. **[3]**

You are now given that A is the point $(11, 12, -14)$ and B is the point $(9, 13, 6)$.

(b) Given that the coordinates of C have the form $(2p, p, 1)$, where p is a constant, determine the coordinates of the possible positions of C. **[6]**

END OF QUESTION PAPER